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Some remarks on topological 4D-gravity

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Abstract

We show that the method of Wu [J. Geom. Phys. 12 (1993) 205] to study topological 4D-gravity can be understood within a standard method now designed to produce equivariant cohomology classes. Next, this general framework is applied to produce some observables of the topological 4D-gravity. © 1998 Elsevier Science B.V.

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1. Introduction

Since their appearance in 1988 in a famous article of Witten [13], topological field theories have played an important role in theoretical physics as well as in mathematics. Actually, the 1988 article gave a prototype of topological field theories of cohomological type. Witten has recognized that these cohomological field theories are related to equivariant cohomology and more precisely to the so-called Cartan model of equivariant cohomology.

Although cohomological field theories can be described independently of the models used for equivariant cohomology, the construction by Kalkman [9] of the so-called intermediate model [12] is of considerable technical help. In [12], topological Yang–Mills [1,3,13] and topological 2D gravity [4,5] were studied from this point of view. In [2], new representatives of the Thom class of a vector bundle were produced using this general framework.

Wu [14] explained the role of the universal bundle in 4D gravity,[2] and exhibited some observables of the corresponding topological model. We shall explain here how his method can be deduced from the general approach of [12] and which observables are obtained.

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2. From the intermediate to the Weil model of equivariant cohomology

In [12] it was explained how one can generate representatives of equivariant cohomology classes using an idea of [6] which benefits from Kalkman's construction [9] as follows: let us assume that $\mathcal{M}$ is a smooth manifold with a smooth $\mathcal{G}$-action for some connected Lie group $\mathcal{G}$ (with Lie algebra Lie $\mathcal{G}$). Let $d_{\mathcal{M}}, i_{\mathcal{M}}, l_{\mathcal{M}}$ be the standard exterior derivative, inner product and Lie derivative on $\mathcal{M}$. The action of $\mathcal{G}$ induces an action of Lie $\mathcal{G}$, and to any $\lambda \in$ Lie $\mathcal{G}$, there corresponds a so-called fundamental vector field $\lambda_{\mathcal{M}}$ on $\mathcal{M}$. The space of forms on $\mathcal{M}$ is denoted by $\Omega(\mathcal{M})$, and its basic elements are those annihilated both by $i_{\mathcal{M}}(\lambda)$ and $l_{\mathcal{M}}(\lambda)$, for any $\lambda \in$ Lie $\mathcal{G}$. We recall that $l_{\mathcal{M}} = [d_{\mathcal{M}}, i_{\mathcal{M}}]_+$.

The Weil algebra $(\mathcal{W}(\mathcal{G}), d_{\mathcal{W}}, i_{\mathcal{W}}, l_{\mathcal{W}})$ of $\mathcal{G}$ is the graded differential algebra generated by the "connection ω" and its "curvature Ω"

$$d_{\mathcal{W}}\omega = \Omega - \tfrac{1}{2}[\omega, \omega], \tag{1}$$

$$d_{\mathcal{W}}\Omega = -[\omega, \Omega], \tag{2}$$

$$i_{\mathcal{W}}(\lambda)\omega = \lambda, \tag{3}$$

$$i_{\mathcal{W}}(\lambda)\Omega = 0, \tag{4}$$

$$l_{\mathcal{W}}(\lambda)\omega = -[\lambda, \omega], \tag{5}$$

$$l_{\mathcal{W}}(\lambda)\Omega = -[\lambda, \Omega], \tag{6}$$

for any $\lambda \in$ Lie $\mathcal{G}$.

Then the equivariant cohomology for the action of $\mathcal{G}$ on $\mathcal{M}$ is the basic cohomology of the graded differential algebra $(\mathcal{W}(\mathcal{G}) \otimes \Omega(\mathcal{M}), d_{\mathcal{W}} + d_{\mathcal{M}}, i_{\mathcal{W}} + i_{\mathcal{M}}, l_{\mathcal{W}} + l_{\mathcal{M}})$. It generates the so-called Weil model of equivariant cohomology.

Now let us consider another Lie group H such that $\mathcal{M}$ is the base space of some principal H-bundle $\mathcal{P}(\mathcal{M}, H)$ on which the action of $\mathcal{G}$ can be lifted. This bundle is also equipped with standard differential operations: $d_{\mathcal{P}}, i_{\mathcal{P}}, l_{\mathcal{P}}$. Then some equivariant cohomology classes can be represented as follows: consider a $\mathcal{G}$-invariant H-connection Γ on $\mathcal{P}$. Extend Γ to $\mathcal{W}(\mathcal{G}) \otimes \Omega(\mathcal{M})$, still denoting it Γ. Since Γ does not depend on ω, it fulfills

$$i_{\mathcal{W}}(\lambda)\Gamma = 0, \tag{7}$$

$$(l_{\mathcal{W}} + l_{\mathcal{P}})(\lambda)\Gamma = 0, \tag{8}$$

for any $\lambda \in$ Lie $\mathcal{G}$. This expresses the basicity of Γ in the so-called intermediate model of equivariant cohomology. In this model, the exterior derivative reads

$$D_{\text{int}} = d_{\mathcal{W}} + d_{\mathcal{P}} + l_{\mathcal{P}}(\omega) - i_{\mathcal{P}}(\Omega) \tag{9}$$

so that

$$D_{\text{int}}\Gamma = d_{\mathcal{P}}\Gamma - i_{\mathcal{P}}(\Omega)\Gamma \tag{10}$$

and the equivariant curvature of Γ in the intermediate model reads

$$R^{\text{eq}}_{\text{int}}(\Gamma, \omega, \Omega) = D_{\text{int}}\Gamma + \tfrac{1}{2}[\Gamma, \Gamma]. \tag{11}$$

It satisfies

$$D_{\mathrm{int}} R^{\mathrm{eq}}_{\mathrm{int}} = [R^{\mathrm{eq}}_{\mathrm{int}}, \Gamma], \tag{12}$$

$$i_{\mathcal{W}}(\lambda) R^{\mathrm{eq}}_{\mathrm{int}} = 0, \tag{13}$$

$$(l_{\mathcal{W}} + l_{\mathcal{P}})\lambda R^{\mathrm{eq}}_{\mathrm{int}} = 0. \tag{14}$$

The H-fibration is eliminated by considering symmetric H-invariant polynomials $I^{\mathrm{eq}}_{\mathrm{int}} = I(R^{\mathrm{eq}}_{\mathrm{int}})$.

To go to the more usual Weil model, we use the Kalkman differential algebra isomorphism $\exp\{i_{\mathcal{P}}(\omega)\}$, thus obtaining

$$(d_{\mathcal{W}} + d_{\mathcal{P}}) I^{\mathrm{eq}}_{\mathrm{W}} = 0, \tag{15}$$

$$(i_{\mathcal{W}} + i_{\mathcal{P}})(\lambda) I^{\mathrm{eq}}_{\mathrm{W}} = 0, \tag{16}$$

$$(l_{\mathcal{W}} + l_{\mathcal{P}})(\lambda) I^{\mathrm{eq}}_{\mathrm{W}} = 0, \tag{17}$$

where $I^{\mathrm{eq}}_{\mathrm{W}} = \exp\{i_{\mathcal{P}}(\omega)\} I^{\mathrm{eq}}_{\mathrm{int}}$. Now since the H-fibration has disappeared, $I^{\mathrm{eq}}_{\mathrm{W}}$ lies in $\mathcal{W}(\mathcal{G}) \otimes \Omega(\mathcal{M})$. Under the assumption that $\mathcal{M}$ is a principal $\mathcal{G}$-bundle over $\mathcal{M}/\mathcal{G}$, we can replace ω and Ω by a $\mathcal{G}$-connection θ and its curvature Θ on $\mathcal{M}$. Cartan's Theorem 3 guarantees that our new representative gives a representative of the same equivariant cohomology class [7,12]. Still denoting this representative by $I^{\mathrm{eq}}_{\mathrm{W}}$, we verify that

$$d_{\mathcal{M}} I^{\mathrm{eq}}_{\mathrm{W}} = 0, \tag{18}$$

$$i_{\mathcal{M}}(\lambda) I^{\mathrm{eq}}_{\mathrm{W}} = 0, \tag{19}$$

$$l_{\mathcal{M}}(\lambda) I^{\mathrm{eq}}_{\mathrm{W}} = 0. \tag{20}$$

Now, we are ready to use this method in topological 4D-gravity.

3. Wu's construction [14] in topological 4D-gravity

Let Σ be a 4D smooth manifold. The fundamental objects in $\mathrm{Gr}^{\mathrm{top}}_4$ are the metrics of Σ and the generators of the Weil algebra of $\mathrm{Diff}_0(\Sigma)$, the connected component of the diffeomorphism group of Σ. The structure equations then read

$$s^{\mathrm{top}} g = \Psi + L^{\mathrm{top}}(\omega) g, \tag{21}$$

$$s^{\mathrm{top}} \Psi = -L^{\mathrm{top}}(\Omega) g + L^{\mathrm{top}}(\omega)\Psi, \tag{22}$$

$$s^{\mathrm{top}} \omega = \Omega - \tfrac{1}{2}[\omega, \omega], \tag{23}$$

$$s^{\mathrm{top}} \Omega = -[\omega, \Omega]. \tag{24}$$

Let us note that the form of these structure equations is universal (i.e. independent of the model we choose). Now, let us apply the precepts of the previous section. The group of diffeomorphisms of Σ plays the role of the gauge group $\mathcal{G}$ over $\mathrm{Met}(\Sigma)$. The H-fibration is obtained by considering the frame bundle over Σ, $F(\Sigma)$,[3] and our final principal

[3] Note that $F(\Sigma)$ is the principal bundle associated to the tangent vector bundle $T\Sigma$ of Σ.

$GL(4,\mathbb{R})$-bundle $\mathcal{P}$ is just $\mathrm{Met}(\Sigma) \times F(\Sigma)$. The $\mathrm{Diff}(\Sigma)$-invariant $GL(4,\mathbb{R})$-connection Γ on $\mathrm{Met}(\Sigma) \times F(\Sigma)$ is given by

$$\Gamma_\mu{}^\lambda = \Gamma^{\mathrm{LC}}(g)_\mu{}^\lambda + \tfrac{1}{2} g^{\lambda\nu} \delta g_{\nu\mu}, \tag{25}$$

where $\Gamma^{\mathrm{LC}}(g)$ is the Levi-Civita connection of $g \in \mathrm{Met}(\Sigma)$, and δ is the exterior derivative on $\mathrm{Met}(\Sigma)$ [4,8].

This $GL(4,\mathbb{R})$-connection is used in the intermediate model. Before going any further, let us notice that in the Weil model, this connection reads

$$\tilde{\Gamma}_\mu{}^\lambda = \Gamma_\mu{}^\lambda - (i_{\mathcal{P}}(\Omega)\Gamma)_\mu{}^\lambda, \tag{26}$$

which is comparable with (2.5) in [14]. Now, the intermediate curvature

$$R^{\mathrm{eq}}_{\mathrm{int}}(\Gamma, \omega, \Omega) = D_{\mathrm{int}}\Gamma - \tfrac{1}{2}[\Gamma, \Gamma] \tag{27}$$

gives the corresponding Weil curvature

$$\begin{aligned} R^{\mathrm{eq}}_{\mathrm{W}}(\Gamma, \omega, \Omega) &= \exp\{i_{\mathcal{P}}(\omega)\} R^{\mathrm{eq}}_{\mathrm{int}}(\Gamma, \omega, \Omega) \\ &= (d_{\mathcal{W}} + d_{\mathcal{P}})\tilde{\Gamma} + \tfrac{1}{2}[\tilde{\Gamma}, \tilde{\Gamma}], \end{aligned} \tag{28}$$

which is of the form (2.6) of Wu [14].

Now, let us construct some observables.

4. Some observables for topological 4D-gravity

In order to generate observables of the theory, we first eliminate the $\mathrm{GL}(4,\mathbb{R})$-fibration. As explained in Section 2 this is achieved by considering symmetric $\mathrm{GL}(4,\mathbb{R})$-invariant polynomials. The Euler class and the Pontrjagin classes generated by $R^{\mathrm{eq}}_{\mathrm{W}}$ are such polynomials [10]. Actually, only the first Pontrjagin class is relevant. [4] Up to normalization factors, those two cohomology classes are given by

$$E^{\mathrm{eq}}_{\mathrm{W}} = \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{g}} g_{\nu\lambda} g_{\sigma\chi} (R^{\mathrm{eq}}_{\mathrm{W}})^\lambda_\mu \wedge (R^{\mathrm{eq}}_{\mathrm{W}})^\chi_\rho, \tag{29}$$

$$P^{\mathrm{eq}}_{\mathrm{W}} = (\delta^\mu_\lambda \delta^\rho_\chi - \delta^\mu_\chi \delta^\rho_\lambda)(R^{\mathrm{eq}}_{\mathrm{W}})^\lambda_\mu \wedge (R^{\mathrm{eq}}_{\mathrm{W}})^\chi_\rho, \tag{30}$$

and decompose into five terms

$$E^{\mathrm{eq}}_{\mathrm{W}} = Q^4_0 + Q^3_1 + Q^2_2 + Q^1_3 + Q^0_4, \tag{31}$$

$$P^{\mathrm{eq}}_{\mathrm{W}} = G^4_0 + G^3_1 + G^2_2 + G^1_3 + G^0_4, \tag{32}$$

[4] The zeroth class is trivially 1 while the second (and the highest) class is the square of the Euler class.

where the upper index refers to the form degree on Met(Σ) while the lower one refers to the form degree on Σ. These expressions are to be compared with (2.9) of [14].[5] Observables extracted from monomials $(E_{\mathrm{W}}^{\mathrm{eq}})^m(P_{\mathrm{W}}^{\mathrm{eq}})^n$.

$$(E_{\mathrm{W}}^{\mathrm{eq}})^m(P_{\mathrm{W}}^{\mathrm{eq}})^n = V_0^{4(m+n)} + V_1^{4(m+n)-1} - V_2^{4(m+n)-2} + V_3^{4(m+n)-3} + V_4^{4(m+n)-4}, \tag{33}$$

with

$$V_0^{4(m+n)} = (Q_0^4)^m(G_0^4)^n, \tag{34}$$

$$V_1^{4(m+n)-1} = n(Q_0^4)^m(G_0^4)^{n-1}G_1^3 + m(Q_0^4)^{m-1}Q_1^3(G_0^4)^n, \tag{35}$$

$$\begin{aligned} V_2^{4(m+n)-2} &= n(Q_0^4)^m(G_0^4)^{n-1}G_2^2 + \frac{n(n-1)}{2}(Q_0^4)^m(G_0^4)^{n-2}(G_1^3)^2 \\ &\quad + mn(Q_0^4)^{m-1}Q_1^3(G_0^4)^{n-1}G_1^3 + m(Q_0^4)^{m-1}Q_2^2(G_0^4)^n \\ &\quad + \frac{m(m-1)}{2}(Q_0^4)^{m-2}Q_2^2Q_1^3(G_0^4)^n, \end{aligned} \tag{36}$$

$$\begin{aligned} V_3^{4(m+n)-3} &= n(Q_0^4)^m(G_0^4)^{n-1}G_3^1 + \frac{n(n-1)}{2}(Q_0^4)^m(G_0^4)^{n-2}G_2^2G_1^3 \\ &\quad + \frac{n(n-1)(n-2)}{6}(Q_0^4)^m(G_0^4)^{n-3}(G_1^3)^3 \\ &\quad + mn(Q_0^4)^{m-1}Q_1^3(Q_0^4)^{n-1}G_2^2 \\ &\quad + m\frac{n(n-1)}{2}(Q_0^4)^{m-1}Q_1^3(G_0^4)^{n-2}(G_1^3)^2 \\ &\quad + mn(Q_0^4)^{m-1}Q_2^2(G_0^4)^{n-1}G_1^3 \\ &\quad + n\frac{m(m-1)}{2}(Q_0^4)^{m-2}(Q_1^3)^2(G_0^4)^{n-1}G_1^3 \\ &\quad + m(Q_0^4)^{m-1}Q_3^1(G_0^4)^n + \frac{m(m-1)}{2}(Q_0^4)^{m-2}Q_2^2Q_1^3(G_0^4)^n \\ &\quad + \frac{m(m-1)(m-2)}{6}(Q_0^4)^{m-3}(Q_1^3)^3(G_0^4)^n, \end{aligned} \tag{37}$$

$$\begin{aligned} V_4^{4(m+n)-4} &= n(Q_0^4)^m(G_0^4)^{n-1}G_0^4 \\ &\quad + \frac{n(n-1)}{2}(Q_0^4)^m(G_0^4)^{n-2}((G_2^2)^2 + G_1^3G_3^1) \\ &\quad + \frac{n(n-1)(n-2)}{6}(Q_0^4)^m(G_0^4)^{n-3}(G_1^3)^2G_2^2 \\ &\quad + \frac{n(n-1)(n-2)(n-3)}{24}(Q_0^4)^m(G_0^4)^{n-4}(G_1^3)^4 \\ &\quad + mn(Q_0^4)^{m-1}Q_1^3(G_0^4)^{n-1}G_3^1 \end{aligned}$$

[5] In earlier references [11] devoted to algebraic studies of topological gravity, one can find similar formulae whose geometrical meaning is given here.

$$
\begin{aligned}
&+m\frac{n(n-1)}{2}(Q_0^4)^{m-1}Q_1^3(G_0^4)^{n-2}G_2^2G_1^3\\
&+m\frac{n(n-1)(n-2)}{6}(Q_0^4)^{m-1}Q_1^3(G_0^4)^{n-3}(G_1^3)^3\\
&+mn(Q_0^4)^{m-1}Q_2^2(G_0^4)^{n-1}G_2^2\\
&+m\frac{n(n-1)}{2}(Q_0^4)^{m-1}Q_2^2(G_0^4)^{n-2}(G_1^3)^2\\
&+n\frac{m(m-1)}{2}(Q_0^4)^{m-2}(Q_1^3)^2(G_0^4)^{n-1}G_2^2\\
&+\frac{mn(m-1)(n-1)}{4}(Q_0^4)^{m-2}(Q_1^3)^2(G_0^4)^{n-2}(G_1^3)^2\\
&+mn(Q_0^4)^{m-1}Q_3^1(G_0^4)^{n-1}G_1^3\\
&+n\frac{m(m-1)}{2}(Q_0^4)^{m-2}Q_2^2Q_1^3(G_0^4)^{n-1}G_1^3\\
&+n\frac{m(m-1)(m-2)}{6}(Q_0^4)^{m-3}Q_1^3(G_0^4)^{n-1}G_1^3\\
&+m(Q_0^4)^{m-1}Q_0^4(G_0^4)^n\\
&+\frac{m(m-1)}{2}(Q_0^4)^{m-2}((Q_2^2)^2+Q_1^3Q_3^1)(G_0^4)^n\\
&+\frac{m(m-1)(m-2)}{6}(Q_0^4)^{m-3}(Q_1^3)^2Q_2^2(G_0^4)^n\\
&+\frac{m(m-1)(m-2)(m-3)}{24}(Q_0^4)^{m-4}(Q_1^3)^4(G_0^4)^n.
\end{aligned}
\tag{38}
$$

Next, we replace ω and Ω by a Diff(Σ)-connection θ and its curvature Θ on Met(Σ). The corresponding forms fulfill the "descent" equations

$$\delta V_p^{4n-p}+d_\Sigma V_{p-1}^{4n-p+1}=0, \tag{39}$$

$$\mathcal{I}(\lambda)V_p^{4n-p}+i_\Sigma(\lambda)V_{p+1}^{4n-p-1}=0, \tag{40}$$

$$\mathcal{L}(\lambda)V_p^{4n-p}+l_\Sigma(\lambda)V_p^{4n-p}=0, \tag{41}$$

where $\mathcal{I}$ and $\mathcal{L}$ are the inner product and Lie derivative on Met(Σ). Finally, we integrate over cycles on Σ to obtain forms on Met(Σ) only

$$V^{4n-p}=\oint_{\gamma_p} V_p^{4n-p}. \tag{42}$$

Exactly as in the 2D-gravity, only

$$V^{4n-4}=\oint_{\Sigma} V_4^{4n-4} \tag{43}$$

defines an equivariant form on $\mathrm{Met}(\Sigma)$. This gives observables of $\mathrm{Gr}_4^{\mathrm{top}}$, which are the analogues of the Mumford invariants appearing in $\mathrm{Gr}_2^{\mathrm{top}}$.

An explicit expression of the Q's and the G's is given in Appendix A.

5. Conclusion

All the work done above can be applied to higher-dimensional gravity theory. Of course this also applies to Yang–Mills topological theory. Nevertheless, in this last case things are much simpler since the gauge group does not act on the space–time manifold Σ, while in gravity theory the diffeomorphism group does.

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Appendix A

It was already shown in [12] that the Weil curvature takes the form

$$\begin{aligned}(R_{\mathrm{W}}^{\mathrm{eq}})^{\nu}_{\mu} = (&R^{\mathrm{LC}} - i_{\Sigma}(\omega)R^{\mathrm{LC}} + \tfrac{1}{2}i_{\Sigma}(\omega)i_{\Sigma}(\omega)R^{\mathrm{LC}} + \tfrac{1}{2}D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}} \\ &- \tfrac{1}{2}i_{\Sigma}(\omega)D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}} - \tfrac{1}{4}\tilde{\psi}\tilde{\psi} + \tfrac{1}{2}D^{\mathrm{LC}} \wedge \tilde{\tilde{\Omega}})^{\nu}_{\mu},\end{aligned} \tag{A.1}$$

where

$$\tilde{\tilde{\gamma}}_{\mu} = (\delta g_{\rho\mu} - l\Sigma(\omega)g_{\rho\mu})\,\mathrm{d}x^{\rho} = \tilde{\gamma}_{\rho\mu}\,\mathrm{d}x^{\rho}, \tag{A.2}$$

$$\tilde{\psi}^{\nu}_{\mu} = g^{\rho\nu}(\delta g_{\rho\mu} - l_{\Sigma}(\omega)g_{\rho\mu}) = g^{\rho\nu}(\tilde{\gamma}_{\rho\mu}) = (g^{-1}\tilde{\gamma})^{\nu}_{\mu}, \tag{A.3}$$

$$(D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}})^{\nu}_{\mu} = g^{\rho\nu}(D\rho^{\mathrm{LC}}\tilde{\tilde{\gamma}}_{\mu} - D\mu^{\mathrm{LC}}\tilde{\tilde{\gamma}}^{\rho}). \tag{A.4}$$

Then, after a "straightforward" algebraic juggle, one finally obtains

$$Q_4^0 = \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{\mathbf{g}}} g_{\nu\lambda}g_{\sigma\chi}(R^{\mathrm{LC}})^{\lambda}_{\mu} \wedge (R^{\mathrm{LC}})^{\chi}_{\rho} = E_{\Sigma}, \tag{A.5}$$

$$Q_3^1 = 2\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{\mathbf{g}}} g_{\nu\lambda}g_{\sigma\chi}(R^{\mathrm{LC}})^{\lambda}_{\mu} \wedge \left(-i_{\Sigma}(\omega)R^{\mathrm{LC}} + \frac{1}{2}D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}}\right)^{\chi}_{\rho}, \tag{A.6}$$

$$\begin{aligned}Q_2^2 = \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{\mathbf{g}}} g_{\nu\lambda}g_{\sigma\chi}[&(i_{\Sigma}(\omega)(R^{\mathrm{LC}})^{\lambda}_{\mu} \wedge (i_{\Sigma}(\omega)R^{\mathrm{LC}})^{\chi}_{\rho} \\ &- 2(i_{\Sigma}(\omega)R^{\mathrm{LC}})^{\lambda}_{\mu} \wedge (D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}})^{\chi}_{\rho} \\ &+ (D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}})^{\lambda}_{\mu} \wedge (D^{\mathrm{LC}} \wedge \tilde{\tilde{\gamma}})^{\chi}_{\rho}\end{aligned}$$

$$+ (R^{\mathrm{LC}})^{\lambda}_{\mu} \wedge (i_{\Sigma}(\omega) i_{\Sigma}(\omega) R^{\mathrm{LC}} - i_{\Sigma}(\omega)(D^{\mathrm{LC}} \wedge \tilde{\bar{\gamma}})$$

$$- \tfrac{1}{2}\tilde{\psi}\tilde{\psi} - D^{\mathrm{LC}} \wedge \tilde{\bar{\Omega}})^{\chi}_{\rho}], \tag{A.7}$$

$$Q^3_1 = \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{\mathbf{g}}} g_{\nu\lambda} g_{\sigma\chi} (i_{\Sigma}(\omega) i_{\Sigma}(\omega) R^{\mathrm{LC}} - i_{\Sigma}(\omega)(D^{\mathrm{LC}} \wedge \tilde{\bar{\gamma}})$$

$$- \tfrac{1}{2}\tilde{\psi}\tilde{\psi} - D^{\mathrm{LC}} \wedge \tilde{\bar{\Omega}})^{\lambda}_{\mu}$$

$$\wedge(-i_{\Sigma}(\omega) R^{\mathrm{LC}} + \tfrac{1}{2} D^{\mathrm{LC}} \wedge \tilde{\bar{\gamma}})^{\chi}_{\rho}, \tag{A.8}$$

$$Q^4_0 = \frac{\varepsilon^{\mu\nu\rho\sigma}}{4\sqrt{\mathbf{g}}} g_{\nu\lambda} g_{\sigma\chi} (i_{\Sigma}(\omega) i_{\Sigma}(\omega) R^{\mathrm{LC}} - i_{\Sigma}(\omega)(D^{\mathrm{LC}} \wedge \tilde{\bar{\gamma}})$$

$$- \tfrac{1}{2}\tilde{\psi}\tilde{\psi} - D^{\mathrm{LC}} \wedge \tilde{\bar{\Omega}})^{\lambda}_{\mu}$$

$$\wedge(i_{\Sigma}(\omega)(i_{\Sigma}(\omega) R^{\mathrm{LC}} - i_{\Sigma}(\omega)(D^{\mathrm{LC}} \wedge \tilde{\bar{\gamma}})$$

$$- \tfrac{1}{2}\tilde{\psi}\tilde{\psi} - D^{\mathrm{LC}} \wedge \tilde{\bar{\Omega}})^{\chi}_{\rho}. \tag{A.9}$$

Finally, the G's are obtained by replacing $(\varepsilon^{\mu\nu\rho\sigma}/\sqrt{\mathbf{g}}) g_{\nu\lambda} g_{\sigma\chi}$ in the Q's by $(\delta^{\mu}_{\lambda}\delta^{\rho}_{\chi} - \delta^{\mu}_{\chi}\delta^{\rho}_{\lambda})$.

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